ANALYSES OF COEFFICIENT OF LATERAL EARTH PRESSURE IN WEDGE-SHAPED GRANULAR MOUND BASED ON JAKY'S (1944) HYPOTHESIS

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ABSTRACT : The relationship between coefficient of lateral earth pressure at rest K_o and friction angle proposed by Jaky (1948) is the most widely-used formula to estimate at-rest pressure in geotechnical engineering practice. This formula is not an empirical formula but a simplified analytical formula based on continuum mechanics. The ratio between lateral to vertical pressures at the symmetrical line of wedge-shaped granular mound reposed at angle of friction in loose state where elastic region is sandwiched by plastic region, is considered to be K_o. Though Jaky's K_o formulation is not well-known, the analytical method was proved to be theoretically acceptable. Still, there is an inconsistency on pressure distribution profile. It has been found that Jaky's assumption is not related to a physical at-rest condition but the result of rotation of principal stress axis from plastic crust to symmetrical line in elastic core. The purpose of this research is basically to review and generalize the formulations of K_o based on Jaky's hypothesis (1944). Hypothesis of quadratic and higher non-linear shear reduction resulted in unrealistic stress distributions, confirming the conclusion made by the earlier researches that Jaky's K_o equation is a coincidental finding. The relative width of elastic region limited in Jaky's assumption was examined. Linear shear reduction with an adjustable relative width of elastic core can improve the analyzed results. The comparison with experimental measurements demonstrated that lateral earth pressures in the mid-plane of granular mound could be estimated by K_o. Also, the relative width of elastic core could be regarded as arching condition and coefficients of lateral earth pressure are the results of arching effects.

KEYWORDS : *Coefficient of lateral earth pressure at rest, Mohr-Coulomb failure criterion, Arching effect, Plasticity theory, Stockpiles.*

1. INTRODUCTION

Analyses of geotechnical engineering problems often require the initial state of stresses in soil mass. For the general state of stresses where the vertical and lateral effective stresses become principal stress acting on principal planes, the coefficient of earth pressure at rest, K_o is frequently used to describe in-situ lateral earth pressure. K_o value is defined as a ratio of horizontal to vertical effective stresses restricted to the particular condition of zero lateral strains., Eq.(1) is the analytical relationship between K_o and the angle of shearing resistance ϕ derived by Jaky (1944). This equation is not an empirical formula but a theoretical solution based on continuum mechanics. Jaky (1948) later simplified to the semi-empirical relationship as shown in Eq.(2). K_o shown in Eq.(1) gives value about 90% of Eq.(2) over a range of ϕ between 10° to 40°. Due to this small difference, Eq.(2) become more familiar afterwards.

$$K_o = \left(1+\frac{2}{3}\sin\phi\right)\frac{1-\sin\phi}{1+\sin\phi} \approx 0.9\left(1-\sin\phi\right) \quad (1)$$

$$K_o = 1-\sin\phi \quad (2)$$

Jaky (1944) formulated K_o equation from a stress analysis in a long wedge-shaped granular heap inclined at a reposed angle ϕ in loose state as shown in Fig. 1. The ratio between lateral to vertical pressures at the symmetrical line of the mound whose elastic region is sandwiched by plastic region is considered to be K_o. In-plane purely frictional Coulomb material with fixed slip planes was assumed in plastic region while the shear stress distribution was supposedly reduced to zero at the center in elastic region.

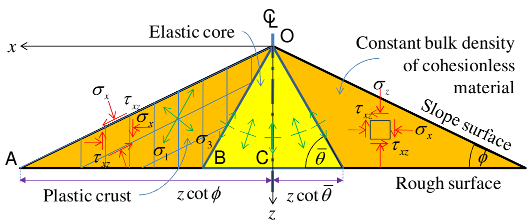

Figure 1 Wedge-shaped granular mound formed by the angle of repose ϕ with horizontal. The vertical z-axis is me-asured vertically downwards from the apex of the mo-und and the horizontal x-axis is measured horizontally outwards.

Though Jaky's K_o formulation is not well-known due to the disadvantage to publishing in Hungarian in 1944, Jaky's assumption and analytical method was critically reviewed by Tschebetarioff (1951), Mesri & Hayat

(1993) and Michalowski (2004, 2005). It was found that the analysis is not associated with one-dimensional strain state but the result of principal stress orientation regulated by the particularly assumed shear stress distribution in granular mound. This assumption does not correspond to K_o physical condition where horizontal boundary of soil mass is infinite. Therefore, Jaky's K_o equation should be exclusively applied to the center line of symmetrical embankments with slope forming the angle of repose with the ground surface. Nevertheless, Jaky's K_o equation is extensively used in practice because the relationship with internal friction angle happens to fit with laboratory measurements for normally consolidated clays and loose granular soils. Michalowski (2005) pointed out that Jaky's analytical method is theoretically acceptable. But stress field across the width of the mound given by Jaky's analysis was found unrealistic when comparing to the experiments on sand piles (Vanel et al., 1999). This verification leads to the conclusion that Jaky's derivation of K_o is a coincidental findings.

It is noted that local constitutive equations are dependent on boundary conditions of the specific problem, thus cannot regarded as a constitutive model of materials. In fact, the local constitutive model based on hypothesis similar to Jaky (1944) was used to explain the marvelous phenomenon of central pressure dip underneath the granular mound. Wittmer et al. (1996, 1997), Cantelaube & Goddard (1997), Cates et al. (1998) and Didwania (2000) also analyzed stress distribution in wedge-shaped granular mound by separating the continuum into elastic and plastic regions. All stress components satisfying the stress equilibrium were continuous across this boundary. The analytical solutions (see Fig.4) reveal the vertical normal stress at the center can exhibit a dip, a peak and a flat by varying the adjustable parameter which is the relative slope of elastic core. The closure remarked that the pressure dip is caused by arching action over elastic core of the heap however arching criterion is still doubted and therefore have not been addressed.

The purpose of this research is basically to review the formulations of K_o based on Jaky's hypothesis. Comparison with fixed principal axis and linear shear reduction models is made. Jaky's model is generalized from linear to higher degree of non-linearity in an attempt to confirm the conclusion made by Michalowski (2005) that Jaky's derivation of K_o is a coincidental result. The limitation of Jaky's hypothesis is discussed and extended. Since the coefficient of lateral earth pressure along the center of the mound can be associated with the relative width of elastic core, the probable linkage of K_o with arching action is explored and validated with experimental measurements. This comprehensive study is expected to be useful for academics by providing initial stress distribution to the analyses of stockpiles and slopes.

2. IDEALIZED GRANULAR WEDGE

The basic theoretical background of Jaky's stress analysis is introduced. In Fig.1, The vertical z-axis is measured vertically downwards from the apex of the mound and the horizontal x-axis is measured horizontally outwards. The symmetrical heap is stabilized by frictional resistance and vertical support of the stiff base with rough surface. The granular wedge is assumed to be a cohesionless material with a constant bulk unit weight γ. The slope OA is inclined at the reposed angle ϕ with horizontal AC. The angle of repose is known as the angle of the maximum slope of the free surface to the horizontal plane. For cohesionless material, the angle of repose is equivalent to the angle of shearing resistance which is independent of the height of the slope surface. Half-width of the continuum of the mound is considered and divided into plastic and elastic regions by the plane OB inclined at $\bar{\theta}$ to the horizontal. The outer region of plastic crust AOB is placed above the inner region of elastic core BOC. Plastic crust is composed of layers of finite thickness with slip planes oriented at ϕ and at $\pi/2$ to horizontal. Therefore, the stress states in region AOB are assumed fully mobilized with fixed direction of the principal stresses. The central core is symmetry along the mid-plane OC which is assumed as the plane of at-rest condition with the direction of major principal stress normal to the horizontal base. Therefore, the stress state in region BOC is assumed elastic with rotating directions of the principal stresses from the elastic-plastic boundary at plane OB to the center line at plane OC. It is required that stress components of plastic and elastic boundary along the plane OB must be connected to maintain the stress continuity. At the slope surface AO, bulk materials are essentially in zero stress state. Moreover, at the mid-plane OC, shear stress must be zero due to the symmetry and absence of frictional horizontal support. For a given depth z, the half-width of the whole heap size is $z\cot\phi$ and the half-width of the whole elastic core is $z\cot\bar{\theta}$.

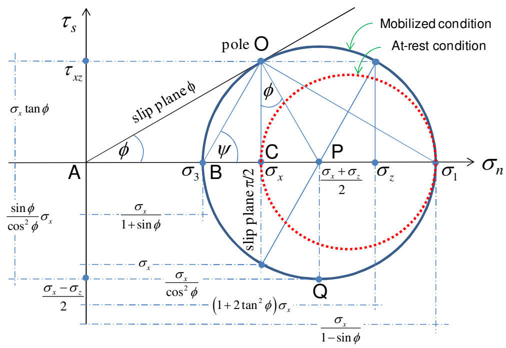

Figure 2 Typical Mohr stress circles for describing the state of stresses in a granular mound. Pole O is determined by the intersection of a slip plane ϕ of slope surface and a conjugate slip plane π/2. The state of stresses of plastic region is represented by the mobilized Mohr circle in which elastic region including the at-rest condition is located inside.

3. CONTINUUM MECHANICS APPROACH

The idealized granular wedge falls in a class of two-dimensional problem. Stress components in (x,z)-plane are given by three independent stress components σ_x, σ_z

and τ_{xz} which must satisfy the equations of equilibrium throughout the wedge with a constant bulk unit weight γ.

$$\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{xz}}{\partial z}=0 \tag{3}$$

$$\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \sigma_z}{\partial z}=\gamma \text{ where } \tau_{zx}=\tau_{xz} \tag{4}$$

First, plastic crust is considered. The state of stresses can be conveniently visualized by Mohr stress circle drawn in Fig.2 with normal stress axis σ_n and tangential shear stress axis τ_s. The pole of the Mohr circle is shown by point O which corresponding to (σ_x, τ_{xz}) on the circle circumference intersected by two slip planes. On σ_n axis, the minor principal stress σ_3 is located by drawing the line joining the pole to the smallest principal stress. Also, the major principal stress σ_1 is located by drawing the line joining the pole to the largest principal stress.

The orientation of major principal stress is represented by the angle ψ measured from σ_n-axis and kept constant throughout region AOB. Using the geometry shown in Fig. 2, ψ can be obtained from a triangle BOP. Jaky (1944) assigned $\overline{\theta}=\psi$, therefore plane OB is parallel to the direction of major principal stress.

$$\psi=\frac{\pi}{4}+\frac{\phi}{2}, \text{ in which } \cot\psi=\frac{\cos\phi}{1+\sin\phi}=\frac{1-\sin\phi}{\cos\phi} \tag{5}$$

According to triangles AOC and COP in Fig.2, shear stress τ_{xz} mobilized at point O due to Morh-Coulomb criterion can be related to normal stresses σ_x and σ_z by,

$$\tau_{xz}=\sigma_x\tan\phi, \quad \frac{\sigma_x+\sigma_z}{2}-\sigma_x=\tau_{xz}\tan\phi \tag{6), (7}$$

As a result, for a given horizontal stress σ_x, a vertical stress σ_z can be determined by substituting Eq.(6) into Eq.(7) with rearrangement to Eq.(8).

$$\sigma_z=\sigma_x+2\tau_{xz}\tan\phi=\left(1+2\tan^2\phi\right)\sigma_x \tag{8}$$

Using Eqs.(6) and (8), in-plane mean stress p and deviatoric stress q in two-dimensional problem are defined as the function with σ_x,

$$p=\frac{\sigma_x+\sigma_z}{2}=\frac{1}{\cos^2\phi}\sigma_x \tag{9}$$

$$q=\sqrt{\left(\frac{\sigma_x-\sigma_z}{2}\right)^2+\tau_{xz}^2}=\frac{\sin\phi}{\cos^2\phi}\sigma_x \tag{10}$$

Stress p and q identifies the center AP and radius PQ of Mohr stress circle in Fig.2 respectively. Consequently, the major and minor principal stresses are determined by,

$$\begin{Bmatrix}\sigma_1\\ \sigma_3\end{Bmatrix}=\begin{Bmatrix}p+q\\ p-q\end{Bmatrix}=\begin{Bmatrix}\dfrac{1}{1-\sin\phi}\\ \dfrac{1}{1+\sin\phi}\end{Bmatrix}\sigma_x \tag{11}$$

As a result, the ratio of the minor to the major principal stresses is constant as given by Eq.(12). This constant is known as Rankine's coefficient of active earth pressure.

$$K_a=\frac{\sigma_3}{\sigma_1}=\frac{1-\sin\phi}{1+\sin\phi} \tag{12}$$

Stress relations of Eqs.(6) and (8) are substituted into equilibrium condition Eqs.(3) and (4), then obtaining the first-order spatial derivatives of σ_x expressed by,

$$\begin{bmatrix}1 & \tan\phi\\ \tan\phi & 1+2\tan^2\phi\end{bmatrix}\cdot\begin{Bmatrix}\dfrac{\partial\sigma_x}{\partial x}\\ \dfrac{\partial\sigma_x}{\partial z}\end{Bmatrix}=\begin{Bmatrix}0\\ \gamma\end{Bmatrix} \tag{13}$$

Components of partial differential equation of horizontal stress with respective to spatial coordinate (x,z) can be obtained by manipulating Eq.(13) to,

$$\begin{Bmatrix}\dfrac{\partial\sigma_x}{\partial x}\\ \dfrac{\partial\sigma_x}{\partial z}\end{Bmatrix}=\frac{\gamma}{1+\tan^2\phi}\begin{Bmatrix}-\tan\phi\\ 1\end{Bmatrix} \tag{14}$$

The differential of σ_x is with respect to (x,z) are given by,

$$d\sigma_x=\frac{\partial\sigma_x}{\partial x}dx+\frac{\partial\sigma_x}{\partial z}dz \tag{15}$$

Integration of $d\sigma_x$ can be carried out due to its simple form. The integral constant c is left after integration.

$$\sigma_x=\frac{\gamma}{1+\tan^2\phi}\left(-\tan\phi\int dx+\int dz\right)+c \tag{16}$$

The state of stresses at the apex of granular wedge is zero, therefore, using the boundary condition defining at x=0,z=0 for σ_x=0, integral constant c can be obtained and stress function σ_x can be determined in closed form.

$$\sigma_x\big|_{x=0,z=0}=0, \text{ hence } c=0 \tag{17}$$

Stress components τ_{xz} and σ_z are obtained by substituting σ_x back to Eqs.(6) and (8) Finally, stress field functions of plastic region either in coordinate parameters (x,z) or shape parameters (x/z,z) are determined.

$$\begin{Bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{Bmatrix}=\gamma z\left(1-\frac{x}{z\cot\phi}\right)\begin{Bmatrix}\cos^2\phi\\ 1+\sin^2\phi\\ \sin\phi\cos\phi\end{Bmatrix} \tag{18}$$

4. JAKY'S MODEL

Elastic core is considered in connection with plastic crust along the plane OB. Therefore, the state of stresses in plastic region AOB and elastic region BOC must be equaled. Since plane OB is inclined at the angle $\overline{\theta}$ with the horizontal direction, the corresponding coordinate $\overline{x}$ can be determined from a given z, or the corresponding coordinate $\overline{z}$ can be determined from a given x vice versa using Eqs.(19) and (20).

$$\overline{x}=z\cot\overline{\theta}, \quad \overline{z}=\frac{x}{\cot\overline{\theta}} \tag{19), (20}$$

The state of stresses along plane OB can be determined by substitution of $x=\overline{x}$ in Eq.(19) into Eq.(18).

$$\begin{Bmatrix}\overline{\sigma}_x\\ \overline{\sigma}_z\\ \overline{\tau}_{xz}\end{Bmatrix}=\begin{Bmatrix}\sigma_x\\ \sigma_z\\ \tau_{xz}\end{Bmatrix}\Bigg|_{x=\overline{x}}=\gamma z\left(1-\frac{\cot\overline{\theta}}{\cot\phi}\right)\begin{Bmatrix}\cos^2\phi\\ 1+\sin^2\phi\\ \sin\phi\cos\phi\end{Bmatrix} \tag{21}$$

Jaky (1944) assumed $\overline{\theta}=\psi$ and chose a quadratic reduction of τ_{xz} along the base to model shear stress distribution from $\overline{\tau}_{xz}$ down to zero at the symmetry plane OC by introducing a local constitutive equation which includes a relative horizontal location $x/\overline{x}$ as follows,

$$\tau_{xz} = \left(\frac{x}{\bar{x}}\right)^2 \bar{\tau}_{xz} = \gamma z \left(\frac{x}{z}\right)^2 \tan\phi(1+\sin\phi) \tag{22}$$

Spatial derivatives of τ_{xz} are given by,

$$\begin{Bmatrix} \frac{\partial \tau_{xz}}{\partial x} \\ \frac{\partial \tau_{xz}}{\partial z} \end{Bmatrix} = \gamma \frac{x}{z} \tan\phi(1+\sin\phi) \begin{Bmatrix} 2 \\ -\frac{x}{z} \end{Bmatrix} \tag{23}$$

Also, stress in elastic core must satisfy equilibrium conditions. σ_x is solved from the partial differential Eq.(3) by partial integration with x using Eq.(23),

$$\sigma_x = -\int \frac{\partial \tau_{xz}}{\partial z} dx + c_z = \frac{\gamma z}{3}\left(\frac{x}{z}\right)^3 \tan\phi(1+\sin\phi) + c_z \tag{24}$$

where c_z is an arbitrary function depending only on z. c_z can be obtained from boundary condition along the plane OB by substituting $x = \bar{x}$ in Eq.(19) to Eq.(25),

$$\sigma_x\big|_{x=\bar{x}} = \bar{\sigma}_x \text{, hence } c_z = \gamma z \frac{1+\frac{2}{3}\sin\phi}{1+\sin\phi}(1-\sin\phi) \tag{25), (26}$$

σ_z is solved from the partial differential Eq.(4) by partial integration with z using Eq.(23),

$$\begin{aligned} \sigma_z &= \int \left(\gamma - \frac{\partial \tau_{zx}}{\partial x}\right) dz + c_x \\ &= \gamma z \left(1 - 2\frac{x}{z}\ln(z)\tan\phi(1+\sin\phi)\right) + c_x \end{aligned} \tag{27}$$

where c_x is an arbitrary function depending only on x. c_x can be obtained from boundary condition along the plane OB by substituting $z = \bar{z}$ in Eq.(20) to Eq.(28),

$$\sigma_z\big|_{z=\bar{z}} = \bar{\sigma}_z \tag{28}$$

$$c_x = \gamma x \tan\phi(1+\sin\phi)\left(2\ln\left(x\frac{1+\sin\phi}{\cos\phi}\right) - \frac{1-\sin\phi}{1+\sin\phi}\right) \tag{29}$$

At the center line OC, state of stresses expressed by Eqs.(24) and (27) have limits when x approaches to zero,

$$\lim_{x\to 0}\sigma_x = \gamma z \frac{1+\frac{2}{3}\sin\phi}{1+\sin\phi}(1-\sin\phi),\ \lim_{x\to 0}\sigma_z = \gamma z \tag{30), (31}$$

Finally, the ratio of horizontal stress to vertical stress K is obtained at the symmetry line of the wedge,

$$K = \lim_{x\to 0}\frac{\sigma_x}{\sigma_z} = \frac{1+\frac{2}{3}\sin\phi}{1+\sin\phi}(1-\sin\phi) \tag{32}$$

Jaky (1944) found that in Eq.(31), σ_z approaches to an equivalent geo-static pressure with increasing depth from ground surface, thus K was assumed to be K_o and later simplified to $K_o = 1 - \sin\phi$. The corresponding state of stress in vertical plane can be referred to point C in Fig. 2.

5. FIXED PRINCIPAL AXIS MODEL

This class of local constitutive equation was initially employed by Witter et al. (1996, 1997) to explain the phenomena of vertical stress dip in the central core. A linear reduction of τ_{xz} along the base was proposed in the similar manner with Jaky (1944). With the elastic-plastic boundary fixed at $\bar{\theta} = \psi$ to the direction of the major principal stress, shear stress distribution in elastic region are given by,

$$\tau_{xz} = \frac{x}{\bar{x}}\bar{\tau}_{xz} = \gamma x \sin\phi \tag{33}$$

Spatial derivatives of τ_{xz} are given below,

$$\frac{\partial \tau_{xz}}{\partial x} = \gamma \sin\phi,\ \frac{\partial \tau_{xz}}{\partial z} = 0 \tag{34), (35}$$

Following the similar procedures of Jaky's model, arbitrary functions and closed-form solutions for σ_x and σ_z obtained from partial integration can be given by the following equations.

$$c_z = \left(\bar{\sigma}_x - \int -\frac{\partial \tau_{xz}}{\partial z} dx\right)\Bigg|_{x=\bar{x}} = \gamma z(1-\sin\phi) \tag{36}$$

$$c_x = \left(\bar{\sigma}_z - \int\left(\gamma - \frac{\partial \tau_{zx}}{\partial x}\right) dz\right)\Bigg|_{z=\bar{z}} = 2\gamma x \frac{\sin^2\phi}{\cos\phi} \tag{37}$$

$$\sigma_x = \gamma z(1-\sin\phi) \tag{38}$$

$$\sigma_z = \gamma z\left(1 - \sin\phi + 2\frac{x}{z}\frac{\sin^2\phi}{\cos\phi}\right) \tag{39}$$

At the center line OC, state of stresses is determined by taking limit on x to 0 in Eqs.(38) and (39). Finally, the ratio K at the symmetry line of the wedge is obtained.

$$\lim_{x\to 0}\sigma_x = \gamma z(1-\sin\phi),\ \lim_{x\to 0}\sigma_z = \gamma z(1-\sin\phi) \tag{40), (41}$$

$$K = \lim_{x\to 0}\frac{\sigma_x}{\sigma_z} = 1 \tag{42}$$

Because ψ is fixed throughout the mound, this solution is termed FPA model (fixed principal axis). FPA model can represent the full arch effect by exhibiting the vertical pressure dip due to the largest shear stress integral induced on the base of granular mound. It is surprised that the value of K is 1, signifying isotropic condition at the central core. This result was formerly discussed by Michalowski (2005) to clarify the reason on why Jaky (1944) particularly chose a quadratic reduction instead of a simple linear reduction.

6. GENERALIZED JAKY'S MODEL

To generalize Jaky's assumption under $\bar{\theta} = \psi$, the shear distribution function is reduced by following a power function as shown below where n is a real number.

$$\tau_{xz} = \left(\frac{x}{\bar{x}}\right)^{1+n} \bar{\tau}_{xz} = \gamma z \tan\phi(1-\sin\phi)\left(\frac{x}{z}\frac{1+\sin\phi}{\cos\phi}\right)^{1+n} \tag{43}$$

The corresponding partial derivatives are obtained by,

$$\frac{\partial \tau_{xz}}{\partial x} = \frac{1+n}{x}\tau_{xz},\ \frac{\partial \tau_{xz}}{\partial z} = -\frac{n}{z}\tau_{xz} \tag{44), (45}$$

Following the similar procedures shown previously, the closed-form solutions for σ_x and σ_z obtained from partial integration can be given by Eqs.(46)-(47). It is found that the solution σ_z has the singularity at $n = 1$.

Thus, the solution of quadratic reduction model obtained in Eqs.(27)-(29) must replace Eq.(47) when $n=1$.

$$\frac{\sigma_x}{\gamma z} = \frac{n}{2+n}\frac{x}{z}\tan\phi\left(1-\sin\phi\right)\left(\frac{x}{z}\frac{1+\sin\phi}{\cos\phi}\right)^{1+n} + \frac{1+\frac{2}{2+n}\sin\phi}{1+\sin\phi}\left(1-\sin\phi\right) \qquad (46)$$

$$\frac{\sigma_z}{\gamma z} = 1-\frac{1+n}{1-n}\left(\frac{x}{z}\frac{1+\sin\phi}{\cos\phi}\right)^{n}\sin\phi + 2\frac{x}{z}\tan\phi\frac{n+\sin\phi}{1-n} \qquad (47)$$

There is an additional singularity when taking the limit of σ_z at the center line. This singularity caused by a term x^n when x approach to 0. Because 0^n is unidentified for $n \le 0$ so, if $n=0$ is employed, this generalized model must be reduced to FPA model. By considering the applicable range of $n \ge 0$, the ratio K is obtained as the conditional function shown below,

$$K = \lim_{x\to 0}\frac{\sigma_x}{\sigma_z} = \begin{cases} 1 & \text{if} \quad n=0 \\ \dfrac{1+\frac{2}{2+n}\sin\phi}{1+\sin\phi}\left(1-\sin\phi\right) & \text{if} \quad n>0 \end{cases} \qquad (48)$$

According to Table 1, despite the unacceptable $K=1$ is obtained when $n=0$, it is found that as n approaches to 0, the expression of K is favorably moving towards $1-\sin\phi$. However, the resulted stress distributions illustrated in Fig.3 by varying n values looks unrealistic due to appearance of unreliable local minimum of vertical pressure found between the center line and elastic-plastic boundary. One may arrive to the conclusion that all shear reduction models including FPA and quadratic shear reduction models are not appropriate. Therefore, the hypothesis proposed by Jaky (1944) are theoretically acceptable in admissible stress fields but unreliable in stress distributions as pointed out by Michalowski (2005) with the conclusion stating that Jaky's derivation of K_o is coincidental.

Table 1 Influence of power degree to coefficient of lateral earth pressure based on nonlinear reduction of shear stress

n	K	Note
∞	$\dfrac{1-\sin\phi}{1+\sin\phi}$	active condition
1	$\left(1+\frac{2}{3}\sin\phi\right)\dfrac{1-\sin\phi}{1+\sin\phi}$	Jaky (1944)'s K_o
1/2	$\left(1+\frac{4}{5}\sin\phi\right)\dfrac{1-\sin\phi}{1+\sin\phi}$	
1/4	$\left(1+\frac{8}{9}\sin\phi\right)\dfrac{1-\sin\phi}{1+\sin\phi}$	
$1/\infty$	$1-\sin\phi$	Jaky (1948)'s K_o
0	1	isotropic condition

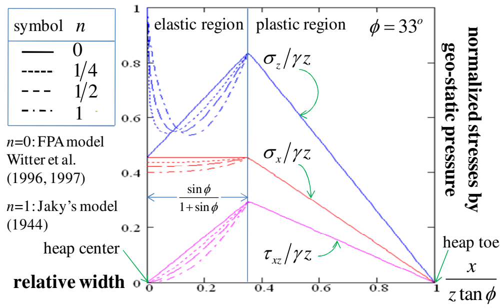

Figure 3 Unrealistic stress distributions results from non-linear shear reduction based on Jaky's (1944) hypothesis. D-espite of $K \to 1-\sin\phi$ as $n \to 0$, unreliable local m-inimum of vertical pressures are appeared for $n<0$ while the isotropic condition where $K=1$ is obtained for $n=0$.

7. LINEAR SHEAR REDUCTION MODEL

FPA model was later extended and verified by Catelaube and Goddard (1997), Cates et al. (1998) and Didwania (2000). Shear stress in elastic core is linearly reduced along horizontal distant from plastic boundary to zero at the center but the relative width of elastic core characterized by the angle $\bar{\theta}$ is not necessarily fixed with the angle ψ of the major principal stress direction.

$$\tau_{xz} = \frac{x}{\bar{x}}\bar{\tau}_{xz} = \gamma z\frac{s}{\bar{s}}\left(1-\bar{s}\right)\sin\phi\cos\phi \qquad (49)$$

Concerning with the solutions obtained from all models, it is evident that all stress components are independent of scale effect and able to express in terms of shape parameters $x/z, z$. For convenience, the relative slope variable s, the relative slope $\bar{s}$ of elastic-plastic boundary are introduced and replaced $x/\bar{x}$ in Eq.(49).

$$s = \frac{x}{z\cot\phi}, \quad \bar{s} = \frac{\bar{x}}{z\cot\phi} = \frac{\cot\bar{\theta}}{\cot\phi} \qquad (50), (51)$$

Following the similar procedures of FPA model, stress field functions are obtained with expressions at the central core by the following equations.

$$\sigma_x = \gamma z\left(1-\bar{s}\right)\cos^2\phi \qquad (52)$$

$$\sigma_z = \gamma z\left(1-s-\left(\frac{1}{\bar{s}}-1\right)\left(1-\left(\frac{1}{\bar{s}}+1\right)s\right)\sin^2\phi\right) \qquad (53)$$

$$\lim_{s\to 0}\sigma_x = \gamma z\left(1-\bar{s}\right)\cos^2\phi \qquad (54)$$

$$\lim_{s\to 0}\sigma_z = \gamma z\left(1-\left(\frac{1}{\bar{s}}-1\right)\sin^2\phi\right) \qquad (55)$$

The coefficient K is determined by a limit of σ_x/σ_z when s approaches to 0 as shown below. It is found that K is depended on value of $\bar{s}$ which is considered as an adjustable parameter of this model.

$$K = \lim_{s\to 0}\frac{\sigma_x}{\sigma_z} = \frac{\bar{s}\left(1-\bar{s}\right)\cos^2\phi}{\bar{s}-\left(1-\bar{s}\right)\sin^2\phi} \qquad (56)$$

Various stress distributions can be obtained by varying $\bar{s}$ or K in Eq.(56). If $\bar{s}$ is specified by taking $\bar{\theta}$ equal to ψ, then the model is reduced to FPA model with $K=1$. If $\bar{s}$ is selected to maintain the constant distribution of σ_z along the horizontal direction, then the model is reduced to BCC model (Bouchaud, Cates and Claudin, 1995) with $K = K_w$ (Krynine (1945)'s wall coefficient). K_w is a ratio of the lateral to vertical pressure in a vertical plane of failure. As shown in Eq.(57), K_w gives higher value than Jaky (1948)'s K_o.

$$K_w = \frac{1}{1+2\tan^2\phi} = \frac{\cos^2\phi}{1+\sin^2\phi} = \frac{1-\sin^2\phi}{1+\sin^2\phi} \tag{57}$$

If K is taken to K_a, then the simplified solution of IFE (incipient failure everywhere) model (Sokolovskii, 1965) can be obtained. If $K = 1-\sin\phi$ is selected, then the stress distribution can be illustrated. Four cases of the explained conditions of K with the specified expressions of $\bar{s}$ are summarized in Table 2. In Fig.4, the variations of stress distribution due to $\bar{s}$ are illustrated and compared with experiments conducted by Vanel et al. (1999). It is found that though K should not be related with K_o, the reasonable stress distributions under $K = K_o = 1-\sin\phi$ can be demonstrated thru this model. So, difference of K could be the result of arching effect and parameter $\bar{s}$ could be regarded as arching condition.

Table 2 The elastic-plastic boundary location determined by various coefficients of lateral earth pressure based on linear shear reduction model

case	K	$\bar{s}$
(1)	$\dfrac{1-\sin\phi}{1+\sin\phi}$	$\dfrac{\sin\phi\left(1+\sqrt{\sin^2\phi+2\sin\phi+2}\right)}{(1+\sin\phi)^2}$
(2)	$1-\sin\phi$	$\dfrac{\sin\phi\left(1-\sin\phi+\sqrt{\sin^2\phi+2\sin\phi+5}\right)}{2(1+\sin\phi)}$
(3)	$\dfrac{1-\sin^2\phi}{1+\sin^2\phi}$	$\dfrac{\sin\phi}{\sqrt{1+\sin^2\phi}}$
(4)	1	$\dfrac{\sin\phi}{1+\sin\phi}$

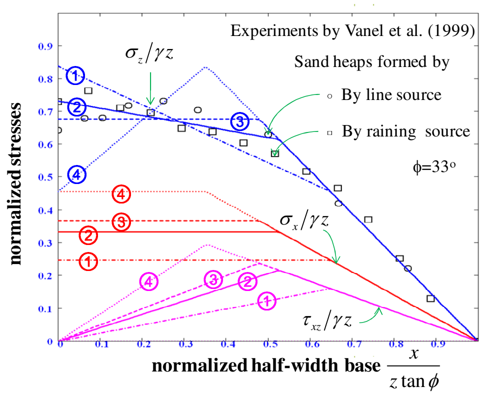

Figure 4 Comparisons between experimental results and stress distributions based on linear shear stress reduction ass-umption which can obtain (1) active earth pressure, (2) Jaky (1948)'s K_o, (3) Krynine (1945)'s K_w, (4) isotro-pic earth pressure.

8. CONCLUSION

A series of generalized solutions aiming to formulate K_o equations based on Jaky (1944)'s hypothesis was presented. These reviews confirm to the conclusion made by Michalowski (2005) that Jaky's K_o is coincidental. The class of shear reduction models is not matched with experiments if Jaky's assumption is followed by fixing the relative width with principal stress direction. Thus, this width should be regarded as a variable which controls arching condition. Then various coefficients of lateral earth pressure are the results of arching effects. Also, this study demonstrated the lateral earth pressure in the mid-plane of the mound can be estimated by K_o.

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