PROBABILISTIC STUDY OF BEARING CAPACITY OF STRIP FOOTING ON COHESIONLESS SOIL

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ABSTRACT: A probabilistic study was carried out to excess the effect of uncertainty in internal friction angle of a cohesionless soil on the bearing capacity of a strip footing. Bearing capacity equation based on limit equilibrium method was used in conjunction with Monte Carlo method to determine the probability distributions of the bearing capacities. Probabilistic study results are shown in term of the distributions of normalized bearing capacity and natural logarithmic of normalized bearing capacity, characterized by their means and standard deviations. These means and standard deviations can be used to calculate probability of bearing capacity failure.

KEYWORDS: BEARING CAPACITY, PROBABILISTIC ANALYSIS, MONTE CARLO SIMULATION, STRIP FOOTING

1. Introduction

Classical bearing capacity equations based on limit equilibrium method, such as the Terzaghi and Meyerhof equations, are traditionally used in calculating bearing capacity of strip footing on a cohesionless soil. In footing design, deterministic soil properties (e.g., average or lowest values of soil friction angle and soil unit weight) are used with conventional factors of safety [1]. Factors of safety are normally selected empirically, i.e., based on past experience or experience with similar engineering structures [2]. Practically, factors of safety have been used in limit equilibrium design to compensate for uncertainties in loads and resistances. Uncertainty in bearing capacity of strip footing may cause by uncertainties in soil properties due to sampling techniques, laboratory test conditions, selection of design parameters from limited samples and laboratory test results, and spatial variability of soil in the field [3].

Uncertainty in bearing capacity of strip footing on a cohesionless soil can be explicitly described in term of the probability distribution, if soil properties are statistically assumed to be normally distributed and can be modeled using probability density function (pdf) characterized by their means and standard deviations. The distribution of the bearing capacity along with its mean and standard deviation can be obtained using a probabilistic method.

In this paper, probabilistic study was conducted to determine the distributions of bearing capacities of a strip footing on a cohesionless soil using Monte Carlo method. Attempts were made to determine the relationships between mean and standard deviation of internal friction angle of soil and means and standard deviations of bearing capacities that can be used to compute the probability of bearing capacity failure of a footing. In addition, an example is used to illustrate the probability of bearing capacity failure and its corresponding factor of safety.

2. Bearing Capacity Model

A general form of bearing capacity equation for strip footing can be expressed as

$$q_{ult} = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma \qquad (1)$$

where q_{ult} = the ultimate bearing capacity of the strip footing, c = cohesion, q = surcharge, γ = unit weight of the soil, B = width of the footing, and N_c, N_q, and N_γ are the bearing capacity factors and can be expressed as

$$N_q = e^{\pi \tan\phi} \tan^2\left(45 + \frac{\phi}{2}\right)$$
$$N_c = (N_q - 1)\cot\phi \qquad (2)$$
$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

where ϕ = internal friction angle of the soil.

For a strip footing on cohesionless soil without surcharge, Eq. 1 can be rewritten as

$$q_{ult} = \frac{1}{2}\gamma BN_\gamma \qquad (3)$$

Assuming that the cohesionless soil is homogeneous within the width and depth of footing, the q_{ult} in Eq. 3 can be normalized by the unit weight of the soil and the width of the footing. Thus, dimensionless normalized bearing capacity (q_N) can be expressed as

$$q_N = \frac{q_{ult}}{\gamma B} = \frac{N_\gamma}{2} \tag{4}$$

3. Probabilistic Study

The process of probabilistic analysis of any engineering quantities (e.g., normalized bearing capacity of strip footing on cohesionless soil) is shown conceptually in Fig. 1. An engineering quantity is calculated using input parameters (e.g., an internal friction angle) treated as random variables that describe their uncertainties. Rather than a single deterministic value for each input parameter, a probability distribution is used to describe the range of possible input values along with its probability of occurrence. When an analysis model (e.g., bearing capacity model) that relates the input parameters to an engineering quantity is used (i.e., Eq. 4), the result is a probability distribution of the engineering quantity [2]. A numerical method such as Monte Carlo method can be used to obtain the distribution of the engineering quantity.

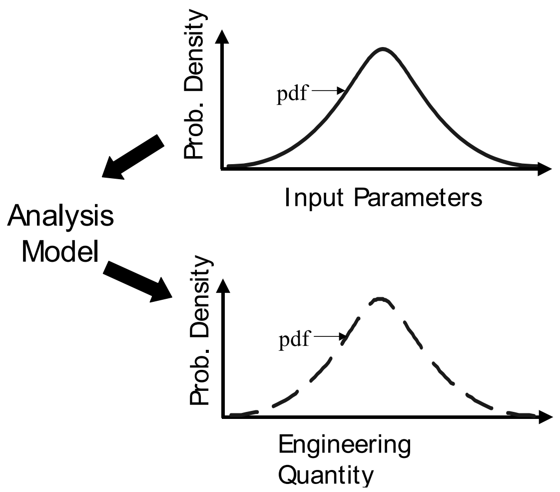

Fig. 1. Schematic of Probabilistic Analysis

Monte Carlo simulation is a statistical tool where the outcome of an event is repeatedly predicted for randomly drawn sets of inputs. The objective of Monte Carlo simulation is to perform enough realizations of a problem such that a probability distribution of the outcomes can be constructed. Numerous samples are taken from probability distribution of the inputs and combined via a model, into a probability distribution of outcome [4].

In this study, to be able to perform Monte Carlo simulation, a random number generator was employed to generate realizations of the internal friction angle from its probability distribution. Only the internal friction angle of a cohesionless soil is treated as a random variable. The distribution of the internal friction angle is assumed to be normal parameterized by μ_ϕ and COV_ϕ as shown by others [7, 8, and 9].

3.1 Generating Realizations of ϕ

The process of generating a realization of ϕ consists of three main steps; 1) generating uniform random numbers, 2) calculating normal random numbers using uniform random numbers just obtained, and 3) calculating a realization of ϕ using normal random numbers.

A very-long-cycle random number generator routine proposed by Wichmann and Hill [5] was used to generate uniform random numbers having uniform distribution characterized by its minimum of 0 and its maximum of 1.

Normal random numbers with mean zero and unit standard deviation were generated from two sets of uniform random numbers using method of Box and Muller [6]. From two sets of uniform random numbers obtained from the Wichmann and Hill routine, U_1 and U_2, the normal random numbers, X_1 and X_2 can be computed as follows:

$$X_1 = \cos(2\pi U_2)\sqrt{-2\ln(U_1)} \tag{5}$$

$$X_2 = \sin(2\pi U_2)\sqrt{-2\ln(U_1)} \tag{6}$$

If only one normal random number is desired, the Box and Muller routine was modified to return a composite normal random number, $X_c(0,1)$:

$$X_c = \frac{X_1 + X_2}{\sqrt{2}} \tag{7}$$

Each realization of internal friction angle (ϕ_i) was computed from composite normal random number ($X_{c,i}$) and mean friction angle (μ_ϕ) and standard deviation of friction angle (σ_ϕ) as:

$$\phi_i = \mu_\phi + X_{c,i}\sigma_\phi \tag{8}$$

Eq. 8 can be written in term of coefficient of variation of internal friction angle (COV_ϕ) as:

$$\phi_i = \mu_\phi\left(1 + X_{c,i}COV_\phi\right) \tag{9}$$

3.2 Monte Carlo Simulations

Monte Carlo simulations were performed to determine the probability distributions of normalized bearing capacity of strip footing on cohesionless soil. As shown by several researchers [4, 7, 8, and 9], the ranges of μ_ϕ and COV_ϕ of natural cohesion soils of 25° to 45° and 5% to 20%, respectively, were used. In each simulation, Eq. 9 was used to generate ϕ_i and Eq. 4 was used to calculate q_N. This process was conducted repeatedly until enough realizations of q_N were obtained. It was found that the number of realizations of 10,000 is appropriate for obtaining the probability distributions of q_N along with its mean and standard deviation.

4. Results and Discussions

4.1 Distributions of ϕ, q_N, and ln q_N

To verify that the generated internal friction angles are normally distributed, a histogram of friction angle generated using μ_ϕ of 30° and COV_ϕ of 10% is shown in Fig. 2a. It is seen that the generated friction angles are normally distributed as a probability density function (pdf) fit to the simulated friction angles is normal (Fig. 2a). Statistical analysis also shows that the average and coefficient of variation of the generated friction angles are 29.99° and 9.99%, which confirms the validity of using random number generators described in this paper in Monte Carlo simulation.

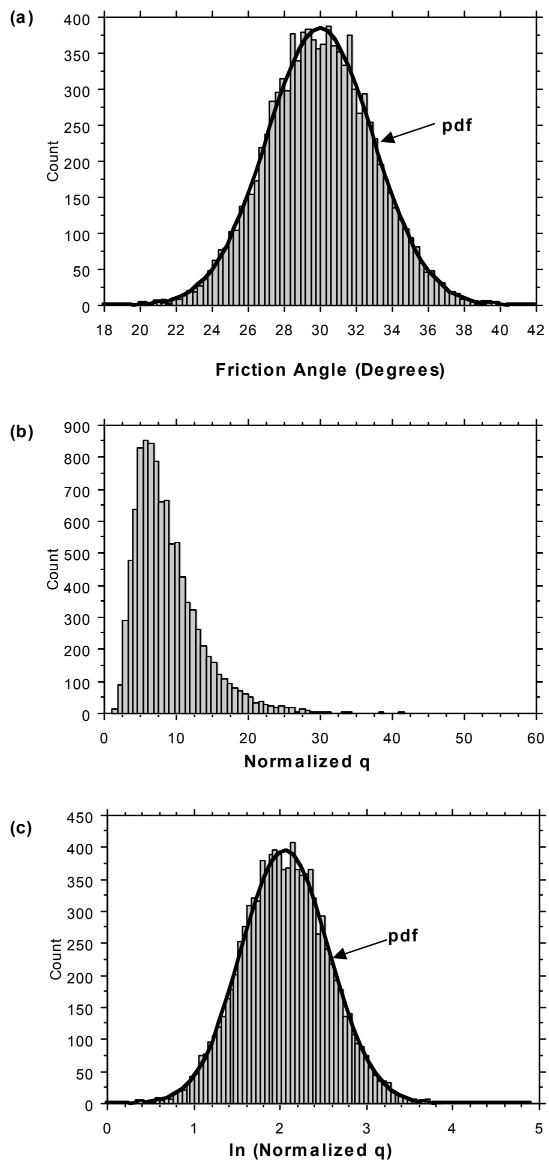

Fig. 2. Histograms Obtained from Monte Carlo Simulations using of μ_ϕ of 30° and COV_ϕ of 10%; a) ϕ, b) q_N, and c) ln q_N.

The histogram of q_N is shown in Fig. 2b. Statistically analysis result shows that the distribution of q_N is lognormally distributed with mean of 8.09 and standard deviation of 9.49. This is not a surprising result because of the fact that in the process of calculating q_N, exponential function is used when computing N_q (Eq. 2). When taking exponential of normal variables, the resulting variables tend to be lognormally distributed.

The distribution of q_N can be used to calculate the probability of failure of the strip footing if the load on the footing is known. However, to use the lognormally distributed parameter is more complicated than using the normally distributed parameter. Moreover, most engineers are more familiar to normal than lognormal distribution. Thus, it is more convenient for most engineers if natural logarithmic of normalized bearing capacity (ln q_N), which is normally distributed, is used instead of q_N.

The distribution of ln q_N was obtained by conducting Monte Carlo simulation using the same process used when obtaining the distribution of q_N but ln q_N which was obtained by taking natural logarithmic on the right side of Eq. 4 was used instead of q_N.

The histogram of ln q_N is shown in Fig. 2c. The pdf fit to simulation results clearly shown that ln q_N is normally distributed with mean of 2.06 and standard deviation of 0.51.

4.2 Relationships between $\mu_{ln\,qN}$, $\sigma_{ln\,qN}$, and μ_ϕ

The relationships between the mean of natural logarithmic of normalized bearing capacity of a strip footing on cohesionless soil ($\mu_{ln\,qN}$) and μ_ϕ are shown in Fig. 3. Data in Fig. 3 were obtained by conducting Monte Carlo simulations with sets of μ_ϕ and COV_ϕ. The μ_ϕ used ranges from 25° to 45°, which practically represents the typical cohesionless soils. The COV_ϕ used ranges from 0% (no uncertainty) to 20% (relatively high uncertainty).

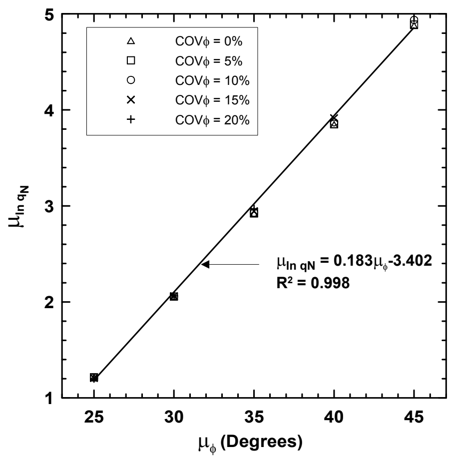

Fig. 3. Relationships between $\mu_{ln\,qN}$ and μ_ϕ

As seen in Fig. 3 that $\mu_{ln\,qN}$ linearly increases with increasing μ_ϕ regardless of the values of COV_ϕ. The linear relationship that relates μ_ϕ to $\mu_{ln\,qN}$ with a coefficient of determination (R^2) of 0.998 is expressed in Eq. (10).

$$\mu_{\ln qN} = 0.183\,\mu_\phi - 3.402 \qquad (10)$$

Along the $\mu_{\ln\,qN}$, the standard deviations of natural logarithmic of normalized bearing capacity of a strip footing on cohesionless soil ($\sigma_{\ln\,qN}$) were also obtained from the simulations. The relationships between $\sigma_{\ln\,qN}$ and μ_ϕ are shown in Fig. 4. As expected, the $\sigma_{\ln\,qN}$ increases with increasing μ_ϕ and COV_ϕ. As μ_ϕ and COV_ϕ increase, realizations of ϕ_ι have more likelihood of having extremely high and extremely low value that widen the tail of the distribution (e.g., Fig. 2a) resulting in expanding of the distribution of the ln q_N (e.g., Fig. 2c) and higher value of $\sigma_{\ln\,qN}$.

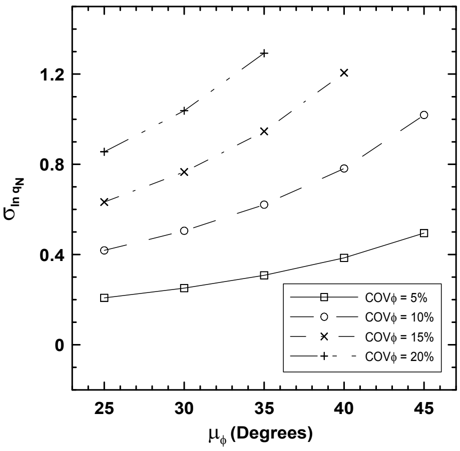

Fig. 4. Relationships between $\sigma_{\ln\,qN}$ and μ_ϕ

4.3 Calculation of Probability of Failure

Uncertainty of the stability of the strip footing can be formally quantified using probability of bearing capacity failure (P_f). P_f is defined by the probability that q_{ult} of the soil underneath the footing is less than the loading pressure (p) which can be expressed as

$$P_f = P[q_{ult} < p] \qquad (11)$$

or it can be expressed in term of ln q_N as

$$P_f = P(\ln q_N < \ln p_N) \qquad (12)$$

where p_N = normalized loading pressure $= \frac{p}{\gamma B}$

P_f can be computed using the procedure demonstrated here. Suppose an engineer would like to calculate the P_f of a 2-m width strip footing located on a cohesionless soil having μ_ϕ of 33°, COV_ϕ of 10%, and γ of 18 kN/m^3. A loading pressure on the footing is 130 kN/m^2, which yields corresponding ln p_N of 1.28. From the given information, $\mu_{\ln\,qN}$ and $\sigma_{\ln\,qN}$ for μ_ϕ of 2.64 and 0.58, respectively, were obtained using Eq. 10 and Fig. 4. Eq. 12 was then used to compute P_f using statistical table of normal distribution provided in any statistics books or using any spreadsheet programs. The computed P_f is 0.01.

To be able to compare the probabilistic approach to deterministic (traditional) approach, a factor of safety (FS) for the bearing capacity was calculated. The FS is defined by ratio of q_{ult} to p (i.e., FS = q_{ult}/p). Using the soil and footing properties given, the q_{ult} computed using Eq. 3 is 471 kN/m^2, while p is 130 kN/m^2. Thus, the corresponding FS that yields P_f of 0.01 is 3.62.

5. Conclusions

Uncertainty of the internal friction angle results in uncertainty of the calculated bearing capacity of a strip footing on a cohesionlesss soil. The uncertainty of bearing capacity was numerically quantified using Monte Carlo method and shown in term of the distribution of natural logarithmic of normalized bearing capacity. This distribution was found to be normal and can be characterized by its mean and standard deviation. The mean and standard deviation of natural logarithmic of normalized bearing capacity can be easily computed using charts that were developed from results of series of simulations. In addition, the mean of the natural logarithmic of normalized bearing capacity is linearly related to the mean of internal friction angle. Probability of bearing capacity failure can be computed if the soil and footing properties and loading pressure are known.

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